【積分計算ドリル】　解答

以下，C は積分定数を表すものとする．

【1】 $\displaystyle\int \tan x dx = \int \frac{-\sin x}{\cos x}\cdot(-1)dx = \boxed{-\log|\cos x| + C}$

【2】 $\displaystyle\int \tan^2 x dx = \int \left(\frac{1}{\cos^2 x} - 1\right)dx = \boxed{\tan x - x + C}$

【3】 $\displaystyle\int \sin 3x \cos 2x dx = \frac{1}{2}\int(\sin 5x + \sin x)dx = \boxed{-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C}$

【4】 $\displaystyle\int \log x dx = \boxed{x\log x - x + C}$

【5】 $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x dx = \int_0^{\frac{\pi}{2}}(1-\sin^2 x)\cos x dx = \left[\sin x - \frac{1}{3}\sin^3 x\right]_0^{\frac{\pi}{2}} = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$

【6】 $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4 x dx = \int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2x}{2}\right)^2 dx = \int_0^{\frac{\pi}{2}} \frac{1 - 2\cos 2x + \frac{1+\cos 4x}{2}}{4}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{1+\frac{1}{2}}{4}dx = \frac{3}{8}\cdot\frac{\pi}{2} = \boxed{\frac{3\pi}{16}}$$

(注：区間 $\left[0,\ \frac{\pi}{2}\right]$ において，$\cos 2x$ は半周期分，$\cos 4x$ は 1 周期分である．)

【7】 $\displaystyle\int_{-1}^0 \frac{1}{x^2+2x+2}dx = \int_{-1}^0 \frac{1}{(x+1)^2+1}dx = \int_0^{\frac{\pi}{4}} \frac{1}{\tan^2\theta + 1}\cdot\frac{1}{\cos^2\theta}d\theta = \boxed{\frac{\pi}{4}}$

($x+1=\tan\theta$ と置換した．)

【8】 $\displaystyle\int_1^2 \frac{1}{x^2+2x}dx = \int_1^2 \frac{1}{2}\left(\frac{1}{x} - \frac{1}{x+2}\right)dx = \frac{1}{2}\left[\log\left|\frac{x}{x+2}\right|\right]_1^2 = \frac{1}{2}\log\left(\frac{2}{4}\cdot\frac{3}{1}\right) = \boxed{\frac{1}{2}\log\frac{3}{2}}$

【9】 $\displaystyle\int_0^1 \frac{1}{x^2+2x+1}dx = \int_0^1 (x+1)^{-2}dx = \Big[-(x+1)^{-1}\Big]_0^1 = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$

【10】 $\displaystyle\int_0^1 (x^2-x+1)^3 dx = \int_0^1 (x^6 - 3x^5 + 6x^4 - 7x^3 + 6x^2 - 3x + 1)dx$

$$= \frac{1}{7} - \frac{1}{2} + \frac{6}{5} - \frac{7}{4} + 2 - \frac{3}{2} + 1 = \frac{1}{7} + \frac{6}{5} - \frac{3}{4} = \frac{20+168-105}{140} = \boxed{\frac{83}{140}}$$