【積分計算ドリル】　解答

以下，C は積分定数を表すものとする．

【1】 $\displaystyle\int \tan 2x dx = \int \frac{-2\sin 2x}{\cos 2x}\cdot\frac{1}{-2}dx = \boxed{-\frac{1}{2}\log|\cos 2x| + C}$

【2】 $\displaystyle\int \frac{1}{\cos^2 x}dx = \boxed{\tan x + C}$

【3】 $\displaystyle\int \cos 4x\cos x dx = \frac{1}{2}\int(\cos 5x + \cos 3x)dx = \boxed{\frac{1}{10}\sin 5x + \frac{1}{6}\sin 3x + C}$

【4】 $\displaystyle\int \log(-x)dx = \boxed{x\log(-x) - x + C}$

【5】 $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 t dt = \int_0^{\frac{\pi}{2}}(\cos^2 t - 1)(-\sin t)dt = \left[\frac{1}{3}\cos^3 t - \cos t\right]_0^{\frac{\pi}{2}} = -\frac{1}{3} + 1 = \boxed{\frac{2}{3}}$

【6】 $\displaystyle\int_0^{\frac{\pi}{2}} \cos^4 t dt = \int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2t}{2}\right)^2 dt = \int_0^{\frac{\pi}{2}} \frac{1 - 2\cos 2t + \dfrac{1+\cos 4t}{2}}{4}dt$

$$= \int_0^{\frac{\pi}{2}} \frac{1 + \dfrac{1}{2}}{4}dx = \frac{3}{8}\cdot\frac{\pi}{2} = \boxed{\frac{3\pi}{16}}$$

$\left(\text{注：区間}\left[0,\ \frac{\pi}{2}\right]\text{において，}\cos 2t\text{ は半周期分，}\cos 4t\text{ は 1 周期分である．}\right)$

【7】 $\displaystyle\int_0^1 \frac{1}{x^2 - 2x + 2}dx = \int_0^1 \frac{1}{(x-1)^2 + 1}dx = \int_{-\frac{\pi}{4}}^0 \frac{1}{\tan^2\theta + 1}\cdot\frac{1}{\cos^2\theta}d\theta = \boxed{\frac{\pi}{4}}$

($x - 1 = \tan\theta$ と置換した.)

【8】 $\displaystyle\int_1^2 \frac{1}{x^2 + x}dx = \int_1^2\left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \left[\log\left|\frac{x}{x+1}\right|\right]_1^2 = \log\left(\frac{2}{3}\cdot\frac{2}{1}\right) = \boxed{\log\frac{4}{3}}$

【9】 $\displaystyle\int_2^3 \frac{1}{4x^2 + 4x + 1}dx = \int_2^3 (2x+1)^{-2}dx = \left[-\frac{1}{2}(2x+1)^{-1}\right]_2^3 = -\frac{1}{2}\left(\frac{1}{7} - \frac{1}{5}\right) = \boxed{\frac{1}{35}}$

【10】 $\displaystyle\int_{-1}^1 (x^2 - x + 3)^2 dx = \int_{-1}^1 (x^4 - 2x^3 + 7x^2 - 6x + 9)dx$

$$= 2\left(\frac{1}{5} + \frac{7}{3} + 9\right) = \frac{2(3 + 35 + 135)}{15} = \boxed{\frac{346}{15}}$$