

以下、 C は積分定数を表すものとする。

$$\text{【1】 } \int \frac{1}{x} \log \frac{1}{x} dx = - \int \log x \cdot \frac{1}{x} dx = \boxed{-\frac{1}{2}(\log x)^2 + C}$$

$$\text{【2】 } \int 2^x(2^x + 1)dx = \int (4^x + 2^x)dx = \frac{4^x}{\log 4} + \frac{2^x}{\log 2} + C = \boxed{\frac{4^x + 2^{x+1}}{2 \log 2} + C}$$

$$\text{【3】 } \int \frac{x^2 - 1}{x^3} dx = \int \left(\frac{1}{x} - x^{-3} \right) dx = \boxed{\log |x| + \frac{1}{2x^2} + C}$$

$$\text{【4】 } \int \sin 2x \sin 3x dx = \int \frac{1}{2}(\cos x - \cos 5x)dx = \boxed{\frac{1}{2} \sin x - \frac{1}{10} \sin 5x + C}$$

$$\begin{aligned} \text{【5】 } \int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx &= \int_0^{\frac{\pi}{2}} \sin^2 x(1 - \sin^2 x) \cos x dx = \int_0^{\frac{\pi}{2}} (\sin^2 x - \sin^4 x) \cos x dx \\ &= \left[\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x \right]_0^{\frac{\pi}{2}} = \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}} \end{aligned}$$

$$\begin{aligned} \text{【6】 } \int_0^{\frac{\pi}{2}} \cos^2 t \sin^2 t dt &= \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} \cdot \frac{1 - \cos 2t}{2} dt \\ &= \frac{1}{4} \int_0^{\frac{\pi}{2}} \left(1 - \frac{1 + \cos 4t}{2} \right) dt = \frac{1}{8} \cdot \frac{\pi}{2} = \boxed{\frac{\pi}{16}} \end{aligned}$$

(注：区間 $\left[0, \frac{\pi}{2}\right]$ において、 $\cos 4t$ は 1 周期分である。)

$$\text{【7】 } \int_{-1}^1 t^3(1-t)^3 dt = 2 \int_0^1 (-t^6 - 3t^4) dt = 2 \left(-\frac{1}{7} - \frac{3}{5} \right) = -\frac{2(5+21)}{35} = \boxed{-\frac{52}{35}}$$

$$\text{【8】 } \int_0^1 t^3(1-t^3) dt = \int_0^1 (t^3 - t^6) dt = \frac{1}{4} - \frac{1}{7} = \boxed{\frac{3}{28}}$$

$$\text{【9】 } \int_0^1 t^2 \sqrt{1-t^3} dt = \int_0^1 (1-t^3)^{\frac{1}{2}} (-3t^2) \cdot \frac{1}{-3} dt = -\frac{1}{3} \left[\frac{2}{3} (1-t^3)^{\frac{3}{2}} \right]_0^1 = \boxed{\frac{2}{9}}$$

$$\text{【10】 } \int_0^1 \sqrt{2-x^2} dx = \img alt="Diagram of a quarter circle in the first quadrant of a Cartesian coordinate system. The circle is centered at the origin (0,0) and has a radius of sqrt(2). The arc is labeled y = sqrt(2-x^2). The x-axis is labeled with 0, 1, and sqrt(2). The y-axis is labeled with 1. A shaded region is shown between the x-axis and the curve from x=0 to x=1. The angle subtended by this region at the origin is 45 degrees." data-bbox="310 780 490 860"/>
 $= \pi(\sqrt{2})^2 \times \frac{45^\circ}{360^\circ} + \frac{1}{2} \cdot 1 \cdot 1 = \boxed{\frac{\pi}{4} + \frac{1}{2}}$$$