【積分計算ドリル】　解答

以下，C は積分定数を表すものとする．

【１】 $\displaystyle\int \frac{1}{x}\log\frac{1}{x}dx = -\int \log x \cdot \frac{1}{x}dx = \boxed{-\frac{1}{2}(\log x)^2 + C}$

【２】 $\displaystyle\int 2^x(2^x+1)dx = \int (4^x + 2^x)dx = \frac{4^x}{\log 4} + \frac{2^x}{\log 2} + C = \boxed{\frac{4^x + 2^{x+1}}{2\log 2} + C}$

【３】 $\displaystyle\int \frac{x^2-1}{x^3}dx = \int \left(\frac{1}{x} - x^{-3}\right)dx = \boxed{\log|x| + \frac{1}{2x^2} + C}$

【４】 $\displaystyle\int \sin 2x \sin 3x dx = \int \frac{1}{2}(\cos x - \cos 5x)dx = \boxed{\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C}$

【５】 $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x\cos^3 x dx = \int_0^{\frac{\pi}{2}} \sin^2 x(1-\sin^2 x)\cos x dx = \int_0^{\frac{\pi}{2}} (\sin^2 x - \sin^4 x)\cos x dx$

$$= \left[\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x\right]_0^{\frac{\pi}{2}} = \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$$

【６】 $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 t\sin^2 t dt = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\cdot\frac{1-\cos 2t}{2}dt$

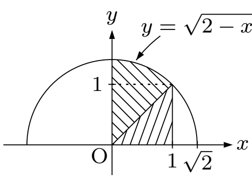
$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1 - \frac{1+\cancel{\cos 4t}}{2}\right)dt = \frac{1}{8}\cdot\frac{\pi}{2} = \boxed{\frac{\pi}{16}}$$

$\left(\text{注：区間}\left[0,\ \frac{\pi}{2}\right]\text{において，}\cos 4t\text{ は 1 周期分である．}\right)$

【７】 $\displaystyle\int_{-1}^1 t^3(1-t)^3dt = 2\int_0^1 (-t^6 - 3t^4)dt = 2\left(-\frac{1}{7} - \frac{3}{5}\right) = -\frac{2(5+21)}{35} = \boxed{-\frac{52}{35}}$

【８】 $\displaystyle\int_0^1 t^3(1-t^3)dt = \int_0^1 (t^3 - t^6)dt = \frac{1}{4} - \frac{1}{7} = \boxed{\frac{3}{28}}$

【９】 $\displaystyle\int_0^1 t^2\sqrt{1-t^3}dt = \int_0^1 (1-t^3)^{\frac{1}{2}}(-3t^2)\cdot\frac{1}{-3}dt = -\frac{1}{3}\left[\frac{2}{3}(1-t^3)^{\frac{3}{2}}\right]_0^1 = \boxed{\frac{2}{9}}$

【10】 $\displaystyle\int_0^1 \sqrt{2-x^2}dx =$ 

$\displaystyle= \pi(\sqrt{2})^2 \times \frac{45^\circ}{360^\circ} + \frac{1}{2}\cdot 1\cdot 1 = \boxed{\frac{\pi}{4} + \frac{1}{2}}$