【積分計算ドリル】　解答

以下，C は積分定数を表すものとする．

【1】$\displaystyle\int \frac{1}{x\log x}dx = \int \frac{1}{\log x}\cdot\frac{1}{x}dx = \boxed{\log|\log x| + C}$

【2】$\displaystyle\int (4^x - 3^{2x})dx = \boxed{\frac{4^x}{\log 4} - \frac{9^x}{\log 9} + C} \left(= \frac{2^{2x-1}}{\log 2} - \frac{3^{2n}}{2\log 3} + C \right)$

【3】$\displaystyle\int \frac{x}{(x-1)^2}dx = \int \frac{(x-1)+1}{(x-1)^2}dx$

$\displaystyle = \int \left\{ \frac{1}{x-1} + (x-1)^{-2} \right\}dx = \boxed{\log|x-1| - \frac{1}{x-1} + C}$

【4】$\displaystyle\int \sin x(1-\cos 10x)dx = \int \left\{ \sin x - \frac{1}{2}(\sin 11x - \sin 9x) \right\}dx$

$\displaystyle = \boxed{-\cos x + \frac{1}{22}\cos 11x - \frac{1}{18}\cos 9x + C}$

【5】$\displaystyle\int_0^\pi \sin^3 u\cos^2 u du = \int_0^\pi (\cos^4 u - \cos^2 u)(-\sin u)du$

$\displaystyle = \left[\frac{1}{5}\cos^5 u - \frac{1}{3}\cos^3 u \right]_0^\pi = \frac{-1-1}{5} - \frac{-1-1}{3} = \boxed{\frac{4}{15}}$

【6】$\displaystyle\int_0^\pi \cos^2 u\sin^2 u du = \int_0^\pi \left(\frac{\sin 2u}{2}\right)^2 du = \frac{1}{8}\int_0^\pi (1-\cos 4u)du = \boxed{\frac{\pi}{8}}$

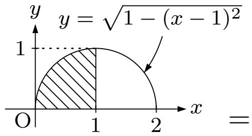
（注：区間 $[0,\ \pi]$ において，$\cos 4u$ は 2 周期分である．）

【7】$\displaystyle\int_{-1}^1 (2u^2-1)^3 du = 2\int_0^1 (8u^6 - 12u^4 + 6u^2 - 1)du$

$\displaystyle = 2\left(\frac{8}{7} - \frac{12}{5} + 2 - 1 \right) = 2\cdot\frac{40-84+35}{35} = \boxed{-\frac{18}{35}}$

【8】$\displaystyle\int_0^1 u^3(1-u^4)^3 du = \frac{1}{-4}\int_0^1 (1-u^4)^3\cdot(-4u^3)du = \frac{1}{-4}\left[\frac{1}{4}(1-u^4)^4 \right]_0^1 = \boxed{\frac{1}{16}}$

【9】$\displaystyle\int_1^2 \frac{u}{\sqrt{u^2+1}}du = \int_1^2 (u^2+1)^{-\frac{1}{2}}\cdot 2u\times\frac{1}{2}du = \frac{1}{2}\left[2(u^2+1)^{\frac{1}{2}} \right]_1^2 = \boxed{\sqrt{5}-\sqrt{2}}$

【10】$\displaystyle\int_0^1 \sqrt{2x-x^2}dx = \int_0^1 \sqrt{1-(x-1)^2}dx =$ 

$\displaystyle = \frac{\pi\cdot 1^2}{4} = \boxed{\frac{\pi}{4}}$