

【積分計算ドリル】 解答

1129

以下, C は積分定数を表すものとする.

$$\begin{aligned} \text{【1】} \int \frac{x}{(x-1)^2} dx &= \int \frac{(x-1)+1}{(x-1)^2} dx = \int \left\{ \frac{1}{x-1} + (x-1)^{-2} \right\} dx \\ &= \boxed{\log|x-1| - \frac{1}{x-1} + C} \end{aligned}$$

$$\text{【2】} \int \cos(\sin x) \cos x dx = \boxed{\sin(\sin x) + C}$$

$$\begin{aligned} \text{【3】} \int \log(x^2 - 1) dx &= \int \{\log|x+1| + \log|x-1|\} dx \\ &= \{(x+1)\log|x+1| - x\} + \{(x-1)\log|x-1| - x\} + C \\ &= \boxed{(x+1)\log|x+1| + (x-1)\log|x-1| - 2x + C} \end{aligned}$$

$$\text{【4】} \int \frac{1}{x^2 \sqrt{x}} dx = \int x^{-\frac{5}{2}} dx = \boxed{-\frac{2}{3}x^{-\frac{3}{2}} + C}$$

$$\begin{aligned} \text{【5】} \int_0^1 \frac{|s^2 - s|}{\sqrt{2}} \cdot s^4 ds &= \frac{1}{\sqrt{2}} \int_0^1 (s - s^2)s^4 ds \quad ([0, 1] \text{において } s^2 - s \leq 0 \text{ である.}) \\ &= \frac{1}{\sqrt{2}} \int_0^1 (x^5 - x^6) ds = \frac{1}{\sqrt{2}} \left(\frac{1}{6} - \frac{1}{7} \right) = \boxed{\frac{1}{42\sqrt{2}}} \end{aligned}$$

$$\begin{aligned} \text{【6】} \int_1^2 t\sqrt{2-t} dt &= \int_1^2 \{(t-2)+2\}(2-t)^{\frac{1}{2}} dt = \int_1^2 \{-(2-t)^{\frac{3}{2}} + 2(2-t)^{\frac{1}{2}}\} dt \\ &= \left[\frac{2}{5}(2-t)^{\frac{5}{2}} - \frac{4}{3}(2-t)^{\frac{3}{2}} \right]_1^2 = \frac{2}{5}(0-1) - \frac{4}{3}(0-1) = \frac{20-6}{15} = \boxed{\frac{14}{15}} \end{aligned}$$

$$\begin{aligned} \text{【7】} \int_0^\pi \sqrt{1 - \cos \frac{x}{2}} dx &= \int_0^\pi \sqrt{2 \sin^2 \frac{x}{4}} dx = \sqrt{2} \int_0^\pi \sin \frac{x}{4} dx \\ &= \sqrt{2} \left[-4 \cos \frac{x}{4} \right]_0^\pi = \boxed{-4 + 4\sqrt{2}} \quad ([0, \pi] \text{において } \sin \frac{x}{4} \geq 0 \text{ である.}) \end{aligned}$$

$$\begin{aligned} \text{【8】} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^3 t + \cos t) \sin^4 t dt &= 2 \int_0^{\frac{\pi}{2}} \cos t \sin^4 t dt = 2 \left[\frac{1}{5} \sin^5 t \right]_0^{\frac{\pi}{2}} = \boxed{\frac{2}{5}} \\ (\sin^7 x \text{ は奇関数であり, } \cos t \sin^4 t \text{ は偶関数である.}) \end{aligned}$$

$$\text{【9】} \int_1^e \log_2 x dx = \int_1^e \frac{\log x}{\log 2} dx = \frac{1}{\log 2} \left[x \log x - x \right]_1^e = \boxed{\frac{1}{\log 2}}$$

$$\begin{aligned} \text{【10】} \int_2^3 \frac{u^2}{u-1} du &= \int_2^3 \frac{(u^2-1)+1}{u-1} du = \int_2^3 \left(u+1 + \frac{1}{u-1} \right) du \\ &= \left[\frac{1}{2}u^2 + u + \log|u-1| \right]_2^3 = \frac{9-4}{2} + 1 + \log 2 = \boxed{\frac{7}{2} + \log 2} \end{aligned}$$