【積分計算ドリル】　解答

1205

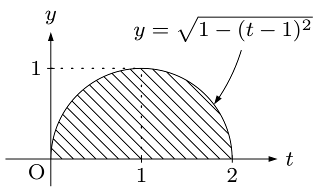
以下，C は積分定数を表すものとする．

【1】$\displaystyle\int \sin x(1-\cos 7x)\,dx = \int \left\{ \sin x - \frac{1}{2}(\sin 8x - \sin 6x) \right\} dx$

$\displaystyle = -\cos x - \frac{1}{2}\left(\frac{-1}{8}\cos 8x - \frac{-1}{6}\cos 6x \right) dx + C = \boxed{-\cos x + \frac{1}{16}\cos 8x + \frac{1}{12}\cos 6x + C}$

【2】$\displaystyle\int \frac{u}{\sqrt{u^2+1}}\,du = \frac{1}{2}\int (u^2+1)^{-\frac{1}{2}} \cdot 2u\,du = \frac{1}{2}\cdot 2(u^2+1)^{\frac{1}{2}} + C = \boxed{\sqrt{u^2+1}+C}$

【3】$\displaystyle\int \log 2x\,dx = x\log 2x - \int \cancel{x} \cdot \frac{\cancel{2}}{\cancel{2x}}\,dx = \boxed{x\log 2x - x + C}$

【4】$\displaystyle\int_0^2 \sqrt{-t^2+2t}\,dt = \int_0^2 \sqrt{1-(t-1)^2}\,dt =$ 

$\displaystyle = \frac{\pi\cdot 1^2}{2} = \boxed{\frac{\pi}{2}}$

【5】$\displaystyle\int_0^1 (x^2-x+1)^2\,dx = \int_0^1 (x^4-2x^3+3x^2-2x+1)\,dx = \frac{1}{5} - \frac{2}{4} + \frac{3}{3} - \frac{2}{2} + 1 = \boxed{\frac{7}{10}}$

【6】$\displaystyle\int_0^{\pi} \cos^4 s\,ds = \int_0^{\pi} \left(\frac{1+\cos 2s}{2} \right)^2 ds = \frac{1}{4}\int_0^{\pi} \left(1 + \cancel{2\cos 2s} + \frac{1+\cancel{\cos 4s}}{2} \right) ds$

$\displaystyle = \frac{1}{4}\int_0^{\pi} \frac{3}{2}\,ds = \frac{1}{4}\cdot\frac{3}{2}\cdot\pi = \boxed{\frac{3}{8}\pi}$

（注：区間 $[0,\ \pi]$ において，$\cos 2s$ は 1 周期分，$\cos 4s$ は 2 周期分である．）

【7】$\displaystyle\int_0^2 3^x(3^x+1)\,dx = \int_0^2 (9^x+3^x)\,dx = \left[\frac{1}{\log 9}9^x + \frac{1}{\log 3}3^x \right]_0^2$

$\displaystyle = \frac{9^2-1}{2\log 3} + \frac{3^2-1}{\log 3} = \frac{80+2\cdot 8}{2\log 3} = \boxed{\frac{48}{\log 3}}$

【8】$\displaystyle\int_1^2 \frac{1}{x^2-2x+2}\,dx = \int_1^2 \frac{1}{(x-1)^2+1}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{\cancel{\tan^2\theta+1}} \cdot \frac{1}{\cancel{\cos^2\theta}}\,d\theta$

$\displaystyle = \int_0^{\frac{\pi}{4}} d\theta = \boxed{\frac{\pi}{4}}$　（$x-1=\tan\theta$ と置換した.）

【9】$\displaystyle\int_0^1 x^2\sqrt{1-x^3}\,dx = -\frac{1}{3}\int_0^1 (1-x^3)^{\frac{1}{2}} \cdot (-3x^2)\,dx = -\frac{1}{3}\left[\frac{2}{3}(1-x^3)^{\frac{3}{2}} \right]_0^1$

$\displaystyle = -\frac{2}{9}(0-1) = \boxed{\frac{2}{9}}$

【10】$\displaystyle\int_1^2 \frac{1}{x}\log\frac{1}{x}\,dx = -\int_1^2 \frac{1}{x}\log x\,dx = -\left[\frac{1}{2}(\log x)^2 \right]_1^2 = \boxed{-\frac{1}{2}(\log 2)^2}$