(1)の【考え方１】で考えてみる。授業では扱っていない、大変は方法です

$$\begin{aligned} Ab_{n+1} - b_n &= A\left\{(n+1)^2 \cdot 3^{(n+1)-1}\right\} - n^2 \cdot 3^{n-1} \\ &= A(n^2+2n+1) \cdot 3^n - n^2 \cdot 3^{n-1} \\ &= \left\{3A(n^2+2n+1) - n^2\right\} \cdot 3^{n-1} \\ &= \left\{(3A-1)n^2 + 6An + 3A\right) \cdot 3^{n-1} \end{aligned}$$

$3A-1=0$ となるように、$A=\dfrac{1}{3}$ と定めてみると

$$\frac{1}{3}b_{n+1} - b_n = (2n+1) \cdot 3^{n-1}$$

$$\therefore\ \frac{1}{3}b_{n+1} - b_n = 2a_n + 3^{n-1}$$

n を k に書き換えると $\dfrac{1}{3}b_{k+1} - b_k = 2a_k + 3^{k-1}$ となり、

$k=1,\ 2,\ \cdots,\ n$ として辺々加えると

$$\sum_{k=1}^{n}\left(\frac{1}{3}b_{k+1} - b_k\right) = \sum_{k=1}^{n}\left(2a_k + 3^{k-1}\right)$$

$$\therefore\ \sum_{k=1}^{n}\left(\frac{1}{3}b_{k+1} - b_k\right) = 2S_n + \sum_{k=1}^{n}3^{k-1} \quad \cdots ②$$

②の左辺は(1)と同様にすると

$$\begin{aligned} (②\text{の左辺}) &= \frac{1}{3}b_{k+1} - \frac{2}{3}T_n - \frac{1}{3}b_1 \\ &= \frac{1}{3}\left\{(n+1)^2 \cdot 3^{(n+1)-1}\right\} - \frac{2}{3}T_n - \frac{1}{3} \cdot 1^2 \cdot 3^{1-1} \\ &= \frac{1}{3}(n+1)^2 \cdot 3^n - \frac{2}{3}T_n - \frac{1}{3} \end{aligned}$$

②の右辺は、S_n に**ス〜タ**の結果を代入し、$\displaystyle\sum_{k=1}^{n}3^{k-1}$ は(1)と同様に計算することで

$$\begin{aligned} (②\text{の左辺}) &= 2 \cdot \frac{(2n-1)3^n + 1}{4} + 1 \cdot \frac{3^n - 1}{3-1} \\ &= \left(\frac{2n-1}{2} + \frac{1}{2}\right)3^n + \left(\frac{1}{2} + \frac{-1}{2}\right) \\ &= n \cdot 3^n \end{aligned}$$

よって、②は次のように表される

$$\frac{1}{3}(n+1)^2 \cdot 3^n - \frac{2}{3}T_n - \frac{1}{3} = n \cdot 3^n$$

$$\therefore\ \frac{2}{3}T_n = \frac{1}{3}(n+1)^2 \cdot 3^n - \frac{1}{3} - n \cdot 3^n$$

$$\begin{aligned} \therefore\ T_n &= \frac{1}{2}(n+1)^2 \cdot 3^n - \frac{1}{2} - \frac{3}{2}n \cdot 3^n \\ &= \left\{\frac{1}{2}(n+1)^2 - \frac{3}{2}n\right\} \cdot 3^n - \frac{1}{2} \\ &= \frac{(n^2 - n + 1) \cdot 3^n - 1}{2} \end{aligned}$$