

Prob 1 (Lv 3.5)

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\left( \cos\theta = \pm \sqrt{\frac{1 + \cos 2\theta}{2}} \right)$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$= \cos\theta (4\cos^2\theta - 3)$$

$$\cos 3\theta = \cos\theta (2\cos 2\theta - 1)$$

$\left(\frac{\pi}{6}\right)$

$\left(\frac{5\pi}{6}\right)$

$$2\cos 2\theta - 1 \neq 0 \text{ t.e.}$$

$$\cos\theta = \frac{\cos 3\theta}{2\cos 2\theta - 1}$$

$\left(\frac{5\pi}{6}\right)$

$\left(\frac{\pi}{6}\right)$

X

$$\text{t.e. } 2\cos 2\theta - 1 = 0 \quad \dots \textcircled{1}$$

$$\cos 3\theta = 0 \quad \dots \textcircled{2}$$

$$\textcircled{1} \text{ t.e. } \cos 2\theta = \frac{1}{2} \quad \text{graph}$$

$$2\theta \in (0, \pi) \text{ t.e. } \theta \in (0, \frac{\pi}{2})$$

$$2\theta = \frac{\pi}{3} \quad \therefore \theta = \frac{\pi}{6} \rightarrow \textcircled{2} \text{ OK}$$

$$\therefore \text{t.e. } \cos\theta = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ t.e. } \left(\frac{\pi}{6}\right) \text{ OK}$$

$$\left(\frac{5\pi}{6}\right) \theta = \frac{\pi}{6} \text{ t.e.}$$

Prob 2

(1) (Lv 2.5)

$$\begin{aligned}
 \int_0^{\frac{\pi}{4}} \frac{x}{\cos^2 x} dx &= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} \cdot x dx \\
 &= \left[ \tan x \cdot x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x dx \\
 &= \left[ x \tan x + \log |\cos x| \right]_0^{\frac{\pi}{4}} \\
 &= \frac{\pi}{4} - \frac{1}{2} \log 2
 \end{aligned}$$

(2) (Lv 3)

$$\begin{aligned}
 \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} &= \int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos^2 x} dx \\
 &= \int_0^{\frac{\pi}{4}} \frac{\cos x}{1 - \sin^2 x} dx \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{4}} \left( \frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} \right) (\sin x)' dx \\
 &= \frac{1}{2} \left[ \log |1 + \sin x| - \log |1 - \sin x| \right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{2} \left[ \log \frac{1 + \sin x}{1 - \sin x} \right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{2} \log \frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} \\
 &= \frac{1}{2} \log \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \\
 &= \frac{1}{2} \log (\sqrt{2} + 1)^2 \\
 &= \log (\sqrt{2} + 1)
 \end{aligned}$$