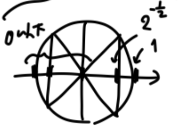
Lv4

$(1+i)^n + (1-i)^n$

$= \left\{\sqrt2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right\}^n + \left\{\sqrt2\left(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4}\right)\right\}^n$

$= 2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4} + \cancel{i\sin\frac{n\pi}{4}} + \cos\frac{-n\pi}{4} + \cancel{i\sin\frac{-n\pi}{4}}\right)$

$= 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$



$(1+i)^n+(1-i)^n>10^{10}$ には $2^{\frac{n}{2}+1}>10^{10}$ が必要

↓ 常対

$\left(\frac{n}{2}+1\right)\log_{10}2>10$

表より 四捨五入

$\log_{10}2 \fallingdotseq 0.3010$

$0.3005 \leqq \log_{10}2 < 0.3015$

$0.31 \overline{)\,20.00}$ → $62.\cdots$
186
140

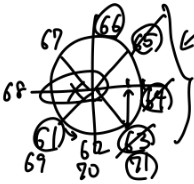
$n > 2\left(\frac{10}{\log_{10}2}-1\right) = \frac{20}{\log_{10}2}-2$

$> \frac{20}{0.3015}-2$

$> \frac{20}{0.31}-2$ ← これぐらい~~荒くても良い?~~ だ×

$= 60.\cdots$

$\therefore n \geqq 61 \equiv 5 \pmod 8$



$n > \frac{20}{0.3015}-2 = 64.\cdots$

$\therefore n \geqq 65$

第1候補 $n=65$

$(1+i)^{65}+(1-i)^{65} = 2^{\frac{65}{2}+1}\times 2^{-\frac{1}{2}}$

$= 2^{33}$

$= 10^{33\log_{10}2}$

$< 10^{33\times 0.3015}$

$< 10^{9.9495}$

$< 10^{10}$ ×

$0.3015 \overline{)\,20.0000}$ → $66.\cdots$
18090
19100
18090
10100

0.3015
× 33
9045
9045
9.9495

$n=66\sim70 \rightarrow \cos\frac{n\pi}{4}<0$ より ×

第2候補 $n=71$

$(1+i)^{71}+(1-i)^{71} = 2^{\frac{71}{2}+1}\times 2^{-\frac{1}{2}}$

$= 2^{36}$

$= 10^{36\times\log_{10}2}$

$> 10^{36\times 0.3005}$

$> 10^{36\times 0.3}$

$= 10^{10.8}$

$> 10^{10}$ OK.

答 $n=\underline{71}$